Modified Distribution Method

The Modified Distribution Method, also known as MODI method or u-v method, which provides a minimum cost solution (optimal solution) to the transportation problem. The following are the steps involved in this method.

Step 1: Find out the basic feasible solution of the transportation problem using any one of the three methods discussed in the previous section.

Step 2: Introduce dual variables corresponding to the row constraints and the column constraints. If there are m origins and n destinations then there will be m+n dual variables. The dual variables corresponding to the row constraints are represented by u_i, i=1,2,…..m whereas the dual variables corresponding to the column constraints are represented by v_j, j=1,2,…..n. The values of the dual variables are calculated from the equation given below $u_i + v_j = c_{ij}$ if $x_{ij} > 0$

Step 3: Any basic feasible solution has m + n -1 $x_{ij} > 0$. Thus, there will be m + n -1 equation to determine m + n dual variables. One of the dual variables can be chosen arbitrarily. It is also to be noted that as the primal constraints are equations, the dual variables are unrestricted in sign.

Step 4: If $x_{ij}=0$, the dual variables calculated in Step 3 are compared with the c_{ij} values of this allocation as $c_{ij} – u_i – v_j$. If al $c_{ij} – u_i – v_j \geq 0$, then by the ***theorem of complementary slackness*** it can be shown that the corresponding solution of the transportation problem is optimum. If one or more $c_{ij} – u_i – v_j < 0$, we select the cell with the least value of $c_{ij} – u_i – v_j$ and allocate as much as possible subject to the row and column constraints. The allocations of the number of adjacent cell are adjusted so that a basic variable becomes non-basic.

Step 5: A fresh set of dual variables are calculated and repeat the entire procedure from Step 1 to Step 5.

Example

Consider the transportation problem given below:

						Supply
	1	9	13	36	51	**50**
	24	12	16	20	1	**100**
	14	33	1	23	26	**150**
Demand	**100**	**70**	**50**	**40**	**40**	**300**

Step 1: First we have to determine the basic feasible solution. The basic feasible solution using least cost method is

x_{11}=50, x_{22}=60, x_{25}=40, x_{31}=50, x_{32}=10, x_{33}=50 and x_{34}=40

Step 2: The dual variables u_1, u_2, u_3 and v_1, v_2, v_3, v_4, v_5 can be calculated from the corresponding c_{ij} values, that is

$u_1+v_1=1$ $u_2+v_2=12$ $u_2+v_5=1$ $u_3+v_1=14$
$u_3+v_2=33$ $u_3+v_3=1$ $u_3+v_4=23$

Step 3: Choose one of the dual variables arbitrarily is zero that is $u_3=0$ as it occurs most often in the above equations. The values of the variables calculated are

$u_1= -13$, $u_2= -21$, $u_3=0$
$v_1=14$, $v_2=33$, $v_3=1$, $v_4=23$, $v_5=22$

Step 4: Now we calculate $c_{ij} – u_i – v_j$ values for all the cells where $x_{ij}=0$ (.e. unallocated cell by the basic feasible solution)
That is

Cell(1,2)= $c_{12}-u_1-v_2 = 9+13-33 = -11$
Cell(1,3)= $c_{13}-u_1-v_3 = 13+13-1 = 25$
Cell(1,4)= $c_{14}-u_1-v_4 = 36+13-23 = 26$
Cell(1,5)= $c_{15}-u_1-v_5 = 51+13-22 = 42$
Cell(2,1)= $c_{21}-u_2-v_1 = 24+21-14 = 31$
Cell(2,3)= $c_{23}-u_2-v_3 = 16+21-1 = 36$
Cell(2,4)= $c_{24}-u_2-v_4 = 20+21-23 = 18$
Cell(3,5)= $c_{35}-u_3-v_5 = 26-0-22 = 4$

Note that in the above calculation all the $c_{ij} – u_i – v_j \geq 0$ except for cell (1, 2) where $c_{12} – u_1 – v_2 = 9+13-33 = -11$.
Thus in the next iteration x_{12} will be a basic variable changing one of the present basic variables non-basic. We also observe that for allocating one unit in cell (1, 2) we have to reduce one unit in cells (3, 2) and (1, 1) and increase one unit in cell (3, 1). The net transportation cost for each unit of such reallocation is

$$-33 -1 + 9 +14 = -11$$

The maximum that can be allocated to cell (1, 2) is 10 otherwise the allocation in the cell (3, 2) will be negative. Thus, the revised basic feasible solution is

x_{11}=40, x_{12}=10, x_{22}=60, x_{25}=40, x_{31}=60, x_{33}=50, x_{34}=40

Unbalanced Transportation Problem

In the previous section we discussed about the balanced transportation problem i.e. the total supply (capacity) at the origins is equal to the total demand (requirement) at the destination. In this section we are going to discuss about the unbalanced transportation problems i.e. when the total supply is not equal to the total demand, which are called as **unbalanced transportation problem**.
In the unbalanced transportation problem if the total supply is more than the total demand then we introduce an additional column which will indicate the surplus supply with transportation cost zero.

Similarly, if the total demand is more than the total supply an additional row is introduced in the transportation table which indicates unsatisfied demand with zero transportation cost.

Example

Consider the following unbalanced transportation problem

Warehouses

Plant	w1	w2	w3	**Supply**
X	20	17	25	400
Y	10	10	20	500
Demand	400	400	500	

In this problem the demand is 1300 whereas the total supply is 900. Thus, we now introduce an additional row with zero transportation cost denoting the unsatisfied demand. So that the modified transportation problem table is as follows:

Warehouses

Plant	w_1	w_2	w_3	**Supply**
X	20	17	25	400
Y	10	10	20	500
Unsatisfied Demand	0	0	0	400
Demand	400	400	500	1300

Now we can solve as balanced problem discussed as in the previous sections.